Chapter-1 : Relations and Functions

1). Show that the relation R defined by $(a,b)\,R\,(c,d) \Rightarrow a+d=b+c$ On the set $N \times N$ is an equivalence relation [2008]

Sol.) $(a,b)\,R\,(c,d) \Rightarrow a+d=b+c$

For reflexive: $(a,b)\,R\,(a,b) \Rightarrow a+b=b+a$, true in N.
Hence, reflexive.

For symmetric: $(a,b)\,R\,(c,d) \Rightarrow a+d=b+c \Rightarrow c+b=d+a$
$\Rightarrow (c,d)\,R\,(a,b) \Rightarrow$ Hence, symmetric.

For transitive:
Let for $(a,b), (c,d), (e,f) \in N \times N$.
$(a,b)\,R\,(c,d)$ and $(c,d)\,R\,(e,f) \Rightarrow (a,b)\,R\,(c,d) \Rightarrow a+d=b+c$ —— (1)
and $(c,d)\,R\,(e,f) \Rightarrow c+f=d+e$ —— (2)
Adding (1) and (2), we get : $a+d+c+f=b+c+d+e$
$\Rightarrow a+f=b+e \Rightarrow (a,b)\,R\,(e,f)$
Hence, transitive. $\Rightarrow$ relation R is an equivalence relation.

2). Let $*$ be a binary operation on N given by $a*b=$ HCF(a, b)
$a,b \in N$. Write the value of $22*4$. [2009]
Sol.) $22*4 = \text{HCF}(22,4) = \underline{\underline{2}}$.

3). Show that the relation S in the set $A=\{x \in Z : 0 \le x \le 12\}$ given by $S=\{(a,b) : a,b \in Z, |a-b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. [2010]

Sol.) Given set $A=\{x \in Z : 0 \le x \le 12\}$
$S=\{(a,b) : a,b \in Z, |a-b| \text{ is divisible by } 4\}$
For reflexive:
For $a \in A$ and $(a,a) \in S \Rightarrow |a-a|$ is divisible by 4.

$\Rightarrow$ 0 is divisible by 4, true.
Hence, reflexive.

For symmetric:

For $a, b \in A$. $(a,b) \in S \Rightarrow |a-b|$ is divisible by 4.
$\Rightarrow |-(b-a)|$ is divisible by 4
$\Rightarrow |b-a|$ is divisible by 4.
$(a,b) \in S \Rightarrow (b,a) \in S$. Hence, symmetric.

For transitive:

For $a, b, c \in A$.
$(a,b) \in S$ and $(b,c) \in S$
$(a,b) \in S$ and $|b-c|$ is divisible by 4
$\Rightarrow |a-b|$ is divisible by 4 and $|b-c|$ is divisible by 4
$\Rightarrow (a-b)$ is divisible by 4 and $(b-c)$ is divisible by 4 —— (1)
$\Rightarrow |a-c| = |(a-b)+(b-c)|$ is divisible by 4. (using (1))
$(a,c) \in S$. Hence, transitive.
As the relation S is reflexive, symmetric and transitive.
Hence, relation S is an equivalence relation
Set of elements related to 1 are $\{1, 5, 9\}$

4) Let $f: R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g: R \to R$ such that $gof = fog = I_R$. [2011]

Sol.) Given $f(x) = y = 10x + 7 \Rightarrow y = 10x + 7$
$\Rightarrow 10x = y - 7 \Rightarrow x = \frac{y-7}{10}$

Let $g(x) = \frac{x-7}{10}$
$\therefore (gof)(x) = g(f(x)) = g(10x+7) = \frac{(10x+7)-7}{10} = x$
and $(fog)(x) = f(g(x)) = f\left(\frac{x-7}{10}\right) = 10\left(\frac{x-7}{10}\right) + 7 = x$
Hence $g(x) = \frac{x-7}{10}$.

5). The binary operation ~~$*: R\times R \to R$~~ on N. is defined as $a*b = \text{lcm}(a,b)$ for all $a, b \in N$. Find $5*7$. [2012]

Sol) $5*7 = \text{lcm}(5,7) = 35$.

6). Consider the binary operations $* : R\times R \to R$ and $o : R\times R \to R$ defined as $a*b = |a-b|$ and $a \circ b = a$ for all $a, b \in R$. Show that $*$ is commutative but not associative, o is associative but not commutative. [2012]

Sol.) Consider binary operation $a*b = |a-b|$

$a*b = |a-b|$ and $b*a = |b-a| = |-(a-b)| = |a-b|$

As $a*b = b*a$ for all $a, b \in R$. Hence $*$ is commutative.

Let $a = 2$, $b = 3$, and $c = 4$.

$(a*b)*c = (2*3)*4 = |2-3|*4$

$= 1*4 = |1-4| = 3$

$a*(b*c) = 2*(3*4) = 2*|3-4| = 2*1 = |2-1| = 1.$

As $(a*b)*c \neq a*(b*c)$.

Hence, $*$ is not associative.

Consider binary operation $a \circ b = a$ for all $a, b \in R$.

$a \circ b = a$ and $b \circ a = b$ for $a, b \in R$.

$a \circ b = a \neq b = b \circ a \Rightarrow o$ is not commutative.

But consider $(a \circ b) \circ c = (a \circ c) = a$

and $a \circ (b \circ c) = a \circ b = \underline{\underline{a}}$.

As $(a \circ b) \circ c = a \circ (b \circ c)$ for all $a, b, c \in R$. Hence o is associat

7). Consider $f: R_+ \to [4, \infty)$ given by $f(x) = x^2+4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$ where R_+ is the set of all non-negative real numbers. [2013]

Sol:) Given $f: R_+ \to [4, \infty)$ as $f(x) = x^2+4$.

For one-one: Let for $x_1, x_2 \in R_+$

$f(x_1) = f(x_2) \Rightarrow x_1^2 + 4 = x_2^2 + 4 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = x_2$ (as x_1, x_2 ...

$\therefore f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. Hence, one-one.

For onto:

Let for $y \in [4, \infty)$, there exists, $x \in R_+$ such that

$y = f(x) \Rightarrow y = x^2+4 \Rightarrow x^2 = y-4 \Rightarrow x = \sqrt{y-4} \in R_+$.

Hence, onto.

As function f is one-one onto. Hence, invertible.

We have, $x = \sqrt{y-4} \Rightarrow f^{-1}(y) = \sqrt{y-4}$

8). If $R = \{(x, y) : x + 2y = 8\}$ is a relation on N, write the range of R. [2014]

Sol.) $R = \{(x, y) : x+2y = 8\}$ is a relation on N.

$\therefore R = \{(2,3), (4,2), (6,1)\}$ $\therefore$ Range $= \{1, 2, 3\}$

9) If the function $f: R \to R$ be given by $f(x) = x^2+2$ and $g: R \to R$ be given by $g(x) = \frac{x}{x-1}$, $x \neq 1$, find fog and gof and hence find $fog(2)$ and $gof(-3)$. [2014]

Sol.). Given $f(x) = x^2+2$ and $g(x) = \frac{x}{x-1}$, $x \neq 1$

$(fog)(x) = f(g(x)) = f\left(\frac{x}{x-1}\right) = \left(\frac{x}{x-1}\right)^2 + 2$

$= \frac{x^2+2(x-1)^2}{(x-1)^2} = \frac{x^2+2x^2-4x+2}{(x-1)^2} = \frac{3x^2-4x+2}{(x-1)^2}$

$(gof)(x) = g(f(x)) = g(x^2+2) = \frac{x^2+2}{x^2+2-1} = \frac{x^2+2}{x^2+1}$

$\therefore (gof)(-3) = \frac{(-3)^2+2}{(-3)^2+1} = \frac{9+2}{9+1} = \frac{11}{10}$ and $(fog)(2) = \frac{3\times 2^2 - 4\times 2+2}{(2-1)^2} = \underline{\underline{6}}$

SAINIK SCHOOL KODAGU

CBSE - Previous years' questions with Solutions
Inverse Trigonometric Functions - Chapter 2

1). Solve for x : $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$ [2009]

Sol.) $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\frac{2}{\sin x}\right)$$

$$\Rightarrow \frac{2\cos x}{\sin^2 x} = \frac{2}{\sin x} \Rightarrow \cot x = 1 \Rightarrow x = \pi/4$$

2). Write the principal value of $\sin^{-1}\left(\sin\frac{4\pi}{5}\right)$. [2010]

Sol.) $\sin^{-1}\left(\sin\frac{4\pi}{5}\right) = \sin^{-1}\left\{\sin\left(\pi - \pi/5\right)\right\} = \sin^{-1}(\sin \pi/5) = \pi/5$

3). Solve for x : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \pi/4$ [2008] & [2010]

Sol.) Given, $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \pi/4$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \pi/4 \Rightarrow \frac{(x-1)(x+2)+(x+1)(x-2)}{(x^2-4)\quad-(x^2-1)} = \tan\pi/4$$

$$\Rightarrow \frac{(x^2+x-2)+(x^2-x-2)}{-3} = 1 \Rightarrow 2x^2-4 = -3$$

$$\Rightarrow 2x^2 = 4-3 = 1 \Rightarrow 2x^2 = 1 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

4). Prove that $\tan^{-1}(1/3) + \tan^{-1}(1/5) + \tan^{-1}(1/7) + \tan^{-1}(1/8) = \pi/4$.

[2010] & [2008]

Sol.) $\left[\tan^{-1}(1/3) + \tan^{-1}(1/5)\right] + \left[\tan^{-1}(1/7) + \tan^{-1}(1/8)\right] = \tan^{-1}\left[\dfrac{1/3 + 1/5}{1 - 1/15}\right] + \tan^{-1}\left[\dfrac{1/7 + 1/8}{1 - 1/56}\right]$

$= \tan^{-1}\left(\dfrac{8}{14}\right) + \tan^{-1}\left[\dfrac{15}{55}\right] = \tan^{-1}\left(\dfrac{4}{7}\right) + \tan^{-1}(3/11) = \tan^{-1}\left[\dfrac{\frac{4}{7} + \frac{3}{11}}{1 - \frac{12}{77}}\right]$

$= \tan^{-1}\left[\dfrac{44 + 21}{77 - 12}\right] = \tan^{-1}\left[\dfrac{65}{65}\right] = \tan^{-1}\boxed{1} = \pi/4$

5). Write the principal value of $\cos^{-1}(\cos 7\pi/6)$

[2011] & [2008]

Sol.) $\cos^{-1}\left(\cos \dfrac{7\pi}{6}\right) = \cos^{-1}\left(2\pi - \dfrac{7\pi}{6}\right) = \cos^{-1}(\cos 5\pi/6) = 5\pi/6$

6). Using principal value evaluate the following:

$\cos^{-1}\left(\cos \dfrac{2\pi}{3}\right) + \sin^{-1}\left(\sin \dfrac{2\pi}{3}\right) = \dfrac{2\pi}{3} + \sin^{-1}(\sin(\pi - \pi/3))$

$= 2\pi/3 + \sin^{-1}(\sin \pi/3) = 2\pi/3 + \pi/3 = \pi.$

P.T.O.

7). Prove the following: $\cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\dfrac{x}{2},\ x\in\left[0,\dfrac{\pi}{4}\right]$ [2011] [20...]

Sol) $\cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\cot^{-1}\left[\dfrac{\sqrt{(\cos x/2+\sin x/2)^2}+\sqrt{(\cos x/2-\sin x/2)^2}}{\sqrt{(\cos x/2+\sin x/2)^2}-\sqrt{(\cos x/2-\sin x/2)^2}}\right]$

$=\cot^{-1}\left[\dfrac{\cos x/2+\sin x/2+\cos x/2-\sin x/2}{\cos x/2+\sin x/2-\cos x/2+\sin x/2}\right]=\cot^{-1}\left[\dfrac{2\cos x/2}{2\sin x/2}\right]=\cot^{-1}(\cot x/2)$

$=x/2$

8). $2\tan^{-1}(1/2)+\tan^{-1}(1/7)=\tan^{-1}\left(\dfrac{31}{17}\right)$ [2011]

Here $2\tan^{-1}(1/2)=\tan^{-1}\left[\dfrac{2(1/2)}{1-(1/2)^2}\right]=\tan^{-1}\left(\dfrac{1}{1-1/4}\right)=\tan^{-1}\left(\dfrac{4}{3}\right)$

Now, $2\tan^{-1}(1/2)+\tan^{-1}(1/7)=\tan^{-1}\left[\dfrac{4/3+1/7}{1-4/3\cdot1/7}\right]=\tan^{-1}\left[\dfrac{\frac{28+3}{21}}{\frac{21-4}{21}}\right]$

$=\tan^{-1}\left(\dfrac{31}{17}\right)$

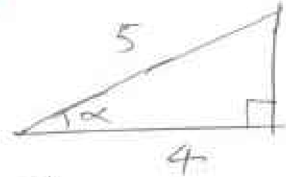
9). Find the principal value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$. [2012]

Sol.) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \pi/3 - (\pi - \pi/3) = \underline{\underline{-\pi/3}}$

10). Prove the following:

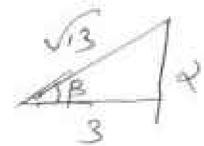
$$\cos\left[\sin^{-1}(3/5) + \cot^{-1}(3/2)\right] = \frac{6}{5\sqrt{13}}$$ [2012]

Sol). In L.H.S., let $\alpha = \sin^{-1} 3/5$ and $\beta = \cot^{-1} 3/2$



$\Rightarrow \sin\alpha = 3/5$ and $\cot\beta = 3/2$

$\Rightarrow \cos\alpha = \frac{4}{5}$ and $\sin\beta = \frac{2}{\sqrt{13}}$ and $\cos\beta = \frac{3}{\sqrt{13}}$



$\therefore$ L.H.S. $= \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \frac{4}{5}\times\frac{3}{\sqrt{13}} - \frac{3}{5}\times\frac{2}{\sqrt{13}}$

$= \frac{12-6}{5\sqrt{13}} = \frac{6}{5\sqrt{13}} = \underline{\underline{\text{R.H.S.}}}$

11). Write the principal value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$ [2013]

Sol.) $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3}) = \frac{\pi}{3} - \frac{5\pi}{6} = \frac{-\pi}{2}$.

12). Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ [2013]

Sol.) Consider $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \tan^{-1}\left[2\sin(2\cdot\pi/6)\right] = \tan^{-1}\left[2\sin\pi/3\right]$

$= \tan^{-1}\left(2\cdot\frac{\sqrt{3}}{2}\right) = \tan^{-1}(\sqrt{3}) = \underline{\underline{\pi/3}}$

13). If $\tan^{-1}x + \tan^{-1}y = \pi/4$, $xy < 1$, then write the value of $x+y+xy$ [2014]

Sol.) $\tan^{-1}x + \tan^{-1}y = \pi/4 \Rightarrow \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \pi/4 \Rightarrow \frac{x+y}{1-xy} = \tan\pi/4 = 1$

$\Rightarrow x+y = 1-xy \Rightarrow x+y+xy = 1$ //

SAINIK SCHOOL KODAGU

CBSE - Previous Years' questions of class XII

Chapter-3 : MATRICES

1). Using elementary transformations, find the inverse of the following matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ [2005]

Sol.) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$; we have $A = IA$

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$ $\left[\because R_2 \to R_2 - 2R_1, R_3 \to R_3 + 2R_1 \right.$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$ $\left[\because R_2 \to R_2 - R_3 \right.$

$\Rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & -2 & 2 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$ $\left[\because R_1 \to R_1 - 2R_2 \right.$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$ $\left[R_1 \to R_1 - 3R_3 \right.$

$\therefore A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

2). Find the value of x, if $\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$ [2005]

Sol.) $3x+y=1$; $-y=2$; $2y-x=-5$ and it implies that $x=1$.

3). Obtain the inverse of the following matrix, using elementary operations: $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$ [2005]

Sol.) Let $A = IA \Rightarrow \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A$